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Superstrings with boundary fermions

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Abstract: The Green-Schwarz action for an open superstring with additional boundary fermions, representing Chan-Paton factors, is studied at the classical level. The boundary geometry is described by a bundle $\widehat{M}$, with fermionic fibres, over the super worldvolume M of a D-brane together with a map from $\widehat{M}$ into the $N = 2$, $D = 10$ target superspace $\underline{M}$. This geometry is constrained by the requirement of kappa-symmetry on the boundary together with the use of the equations of motion for the fermions. There are two constraints which are formally similar to those that arise in the abelian case but which differ because of the dependence on the additional coordinates. The model, when quantised, would be a candidate for a fully kappa-symmetric theory of a stack of coincident D-branes including a non-abelian Born-Infeld sector. The example of the D9-brane in a flat background is studied. The constraints on the non-abelian field strength are shown to be in agreement with those derived from the pure spinor approach to the superstring. A covariant formalism is developed and the problem of quantisation is discussed.

Keywords: Sigma Models, D-branes, Superspaces.

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Contents

1. Introduction

Born-Infeld dynamics has turned out to play a major rôle in string theory. This was first observed in studies of the open string sigma model [1 − 4] but has come to greater prominence in the context of D-branes. As is well-known, the effective lagrangian for a single D-brane in type-II string theory consists of a supersymmetric Dirac-Born-Infeld kinetic term together with an appropriate Wess-Zumino term which involves the RR fields and the worldvolume electromagnetic field strength tensor suitably modifed by the NS two-form. An intriguing problem which is still not fully understood is what the non-abelian version of this theory is. This theory should describe the effective dynamics of a set of coincident D-branes. In this paper we shall outline an approach to this problem based on demanding kappa-symmetry for a Green-Schwarz superstring action with boundary fermions which represent Chan-Paton factors.

Many features of the bosonic terms in the action are known. In the simplest generalisation of the Born-Infeld action to the non-abelian case, proposed in [5], the electromagnetic field strength tensor is replaced by a non-abelian one and the Yang-Mills indices are dealt with by taking the symmetrised trace in each term in the power series expansion of the lagrangian. It is known from analyses of string scattering amplitudes [6] that there are other terms in the effective action of the open string, but the Tseytlin action is expected to provide at least a part of the non-abelian Born-Infeld theory. Dimensional reduction of this action together with T-duality yields a lot of information about terms in the action involving scalar fields and also about the Wess-Zumino term [7]. In particular, one sees the emergence of non-commutativity (the transverse scalars become matrix valued) and the interesting feature that lower-dimensional branes can detect higher-dimensional RR fields. These features were also found from studies of D-branes using matrix models [8, 9].

The results referred to above do not include any fermion fields or make any reference to supersymmetry. Before discussing this point further in the non-abelian case it is perhaps worth pointing out that the effective action for a single D-brane in the Born-Infeld approximation is completely fixed by supersymmetry. This can perhaps be seen most elegantly in the superembedding formalism [10 − 12] where a single constraint on the embedding of the super worldvolume of the brane into the super target space determines the structure of the worldvolume multiplet, the dynamics of this multiplet and implies that the background should be an on-shell supergravity solution [13, 14]. In cases of low codimension one must also impose a constraint on the modified field strength tensor $\mathcal{F}$ [15, 16] which can be derived from the superembedding constraint in the other cases. Both constraints can be derived from an analysis of superstrings ending on branes, either in a hybrid GS approach [17], or completely within the superembedding formalism [18].

There have been various attempts to generalise the above to the non-abelian case. A model with matrix-valued kappa-symmetry was proposed in [19], although it does not seem to be quite right [20]. Actions with a single kappa-symmetry incorporating non-abelian multiplets interacting with the geometry of a brane have been constructed in lower dimensional target superspaces [21 − 23]. There have also been studies of supersymmetric non-abelian Born-Infeld theory from a purely field-theoretic point of view [24 − 26]. The supersymmetrisation of a number of terms known to be present in the effective action of open strings was discussed in components in [27, 28] and in $N = 4$ superfields in [29]. In particular, these studies made contact with the proposal for the purely bosonic sector based on T-duality put forward in [30], and with results derived from scattering amplitude calculations [31 − 33]. Another possible approach to non-abelian Born-Infeld dynamics is via the study of boundary couplings in the RNS sigma model [34, 35].

The subject has also been discussed in the pure spinor approach to string theory. In particular, Berkovits and Pershin derived some terms involving non-abelian fields in the background of a single brane using boundary fermions to represent Chan-Paton factors and working at the classical level with respect to these fields [36] (see also [37]). The idea of using boundary fermions in this way was introduced by [38] and studied further in [39, 40]. In the current paper we shall adopt the philosophy of Berkovits and Pershin but in the context of the Green-Schwarz string. We generalise in several ways. We discuss branes of arbitrary dimension, we allow for arbitrary supergravity backgrounds, we work to all orders in the boundary fermions and we use covariant methods in as far as this is possible. This approach therefore defines a particular approximation to the effective string action with non-abelian fields. In principle, we could obtain a better approximation by quantising the fermions. One might propose the result of this quantisation as a definition of what one means by non-abelian Born-Infeld dynamics for a set of coincident branes. As is well-known, it is not self-evident what this approximation should be because some derivative terms can be replaced with commutators in the non-abelian theory.

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As we shall briefly discuss at the end of the paper, it is not entirely straightforward to carry out the quantisation of the fermions by themselves. The main reason for this is that the geometrical picture underlying the construction is slightly unusual. In the case of a single D-brane one takes the (Green-Schwarz) sigma model field to map into the super worldvolume of the brane on the boundary of the string. In the present case, following Berkovits and Pershin, we shall take the boundary sigma model field to depend on the boundary fermions as well. One way of thinking about this would be to replace the type II target space by a space which has additional fermionic fibres which are localised on the brane. This is a somewhat singular construction but one could view it as the limit of a thickened brane. This would be reasonable on physical grounds since there are supposed to be several branes which coincide. Moreover, the supergravity brane solution also has a natural thickness. However, from the point of view of quantising the model, this picture implies that there is a hidden dependence on the fermions residing in the bulk part of the action. Another way of thinking about this geometry is that it roughly corresponds to the notion of a coherent sheaf, a topic that has been discussed in the boundary fermion picture in the literature [41].

We shall not address the problem of quantisation to any great extent in this paper, postponing a fuller discussion to a later date. However, there are several nice features that can be observed in the classical boundary fermion approximation. In particular, we can keep control of all of the symmetries of the problem in a systematic fashion. These include the one-form gauge symmetry associated with the NS B-field for which we can derive the appropriate modification of the non-abelian field strength tensor which respects this symmetry. We also discuss the diffeomorphisms of the super worldvolume M. Since this is now the base of a larger space, $\widehat{M}$, which includes the additional fermions as fibre coordinates, these diffeomorphisms naturally depend on all of the coordinates. This is required in order for us to be able to gauge away all of the non-physical degrees of freedom and is related to the idea of matrix-valued kappa-symmetry introduced in [19]. These features are all derived systematically from the action in the next section. It is also shown that the requirement of kappa-symmetry on the boundary of the string leads to geometrical conditions on what one might call the generalised superembedding defined by the map from $\widehat{M}$ into the super target space $\underline{M}$. These constraints are remarkably similar to those for a single brane, although it is somewhat more difficult to work out the consequences. In section four we do this for the D9-brane in a flat target space and relate our results explicitly to those of Berkovits and Pershin. In section five we propose a completely covariant formulation of the generalised superembedding. This version is covariant with respect to diffeomorphisms of $\widehat{M}$. In section six we briefly discuss the quantisation problem and give our conclusions.

2. Action with boundary fermions

2.1 Action and variation

The Green-Schwarz action for the superstring is described by a map f_o from a bosonic worldsheet Σ to an $N = 2, D = 10$ target superspace $\underline{M}$. We shall suppose that the

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boundary of the worldsheet, $\partial\Sigma$, maps to a space, $\widehat{M}$, which is a bundle over the super worldvolume, M, of a D-brane, the fibres of this bundle being fermionic. There is a map, $\widehat{f}:\widehat{M}\to\underline{M}$ which extends a superembedding, $f:M\to\underline{M}$, of the brane into the target superspace. The interpretation of this set-up is that the brane is actually the centre-of-mass of a set of coincident D-branes while the fermionic coordinates represent Chan-Paton factors. As we shall see, this allows us to introduce non-abelian gauge fields on the brane.

The worldsheet coordinates are denoted by $y^i, i=0,1$ and the coordinate on the boundary is denoted by t. The coordinates of the superspace M are $z^M=(x^m,\theta^\mu)$ and the coordinate basis one-forms dz^M are related to preferred basis one-forms $E^A=(E^a,E^\alpha)$ by means of the supervielbein, $E^A=dz^M E_M{}^A$. Similar conventions are used for $\underline{M}$, with the difference that all indices are underlined. Primed indices $A'=(a',\alpha')$ denote directions normal to the brane in $\underline{M}$. The coordinates for $\widehat{M}$ are denoted by $z^{\widehat{M}}=(z^M,\eta^{\widehat{\mu}})$, where the index $\widehat{\mu}$, which labels the boundary fermions, can take on n values. When coordinates are used as arguments of functions we shall write $z,\widehat{z},\underline{z}$ for functions on $M,\widehat{M},\underline{M}$ respectively.

The bulk part of the GS action is the same as usual, but we shall allow for a general supergravity background which would in any case be required by kappa-symmetry. In our conventions this action is

$$S_{\rm bulk}=-\int_\Sigma d^2\,y\left(\sqrt{-g}+\frac{1}{2}\epsilon^{ij}B_{ij}\right), \tag{2.1}$$

where g_{ij} is the induced GS metric on Σ,

$$g_{ij}:=E_i{}^{\underline{a}}E_j{}^{\underline{b}}\eta_{\underline{ab}}\,. \tag{2.2}$$

The notation $E_i{}^{\underline{A}}$ denotes the pull-back map from $\underline{M}$ to Σ in a preferred basis,

$$E_i{}^{\underline{A}}:=\partial_i z^{\underline{M}}E_{\underline{M}}{}^{\underline{A}}\,. \tag{2.3}$$

Similarly, B_{ij} denotes the pull-back of the NS two-form potential,

$$B_{ij}:=\partial_j z^{\underline{N}}\partial_i z^{\underline{M}}B_{\underline{MN}}=E_j{}^{\underline{B}}E_i{}^{\underline{A}}B_{\underline{AB}}\,. \tag{2.4}$$

The variation of the bulk action gives

$$\begin{aligned}\delta S_{\rm bulk}=&\int_\Sigma d^2y\left\{\sqrt{-g}g^{ij}\left(\delta z^{\underline{a}}\nabla_i E_{j\underline{a}}+\delta z^{\underline{A}}E_i{}^{\underline{B}}T_{\underline{BA}}{}^{\underline{c}}E_{j\underline{c}}\right)+\frac{1}{2}\delta z^{\underline{A}}\epsilon^{ij}E_i{}^{\underline{B}}E_j{}^{\underline{C}}H_{\underline{CBA}}\right\}+\\&+\int_\Sigma d^2y\,\partial_i\left(-\sqrt{-g}g^{ij}\delta z^{\underline{a}}E_{j\underline{a}}+\epsilon^{ij}\delta z^{\underline{A}}E_j{}^{\underline{B}}B_{\underline{BA}}\right).\end{aligned} \tag{2.5}$$

In this equation $\delta z^{\underline{A}}:=\delta z^{\underline{M}}E_{\underline{M}}{}^{\underline{A}}$ denotes the variation referred to a preferred basis, ∇_i is a covariant derivative with respect to both target space and worldsheet indices, $T_{\underline{AB}}{}^{\underline{C}}$ is the target space torsion and $H=dB$ is the NS three-form field strength. Bosonic target space indices can be raised and lowered by means of $\eta_{\underline{ab}}$. In the rest of the paper we shall only be concerned with the second line of the right-hand-side of this equation which will contribute to the boundary theory. This contribution is

$$\delta S_{\rm bulk-bdry}=\int dt\,n_i\left(-g^{ij}\delta z^{\underline{a}}E_{j\underline{a}}+\frac{\epsilon^{ij}}{\sqrt{-g}}\delta z^{\underline{A}}E_j{}^{\underline{B}}B_{\underline{BA}}\right) \tag{2.6}$$

where n_i denotes a unit normal to $\partial\Sigma$. This can be rewritten as

$$\delta S_{\text{bulk-bdry}} = \int dt \left(-\delta z^{\underline{M}} E_{\underline{M}}{}^{\underline{a}} E_{1\underline{a}} + \delta z^{\underline{M}} \dot{z}^{\underline{N}} B_{\underline{NM}}\right). \tag{2.7}$$

where

$$E_{1\underline{a}} := n_i g^{ij} E_{j\underline{a}}\,. \tag{2.8}$$

In the case of a single D-brane, the mapping would be restricted to the worldvolume of the brane on the boundary. In this case, however, we shall assume that the boundary field takes its values in $\widehat{M}$. Thus, on the boundary, $z^{\underline{M}} = z^{\underline{M}}(\widehat{z}) = z^{\underline{M}}(z^M, \eta^{\widehat{\mu}})$. Taking this into account, we find that (2.7) can be written

$$\delta S_{\text{bulk-bdry}} = \int dt\, \delta z^{\widehat{M}} \partial_{\widehat{M}} z^{\underline{M}} \left(-E_{\underline{M}}{}^{\underline{a}} E_{1\underline{a}} + \dot{z}^{\widehat{N}} \partial_{\widehat{N}} z^{\underline{N}} B_{\underline{NM}}\right). \tag{2.9}$$

The action for the boundary fermions is

$$S_{\text{bdry}} = \int_{\partial\Sigma} \mathcal{A} \tag{2.10}$$

where $\mathcal{A}$ is the pull-back of a one-form potential on $\widehat{M}$. The variation of this action gives

$$\delta S_{\text{bdry}} = -\int_{\partial\Sigma} dt\, \delta z^{\widehat{M}} \dot{z}^{\widehat{N}} (d\mathcal{A})_{\widehat{N}\widehat{M}}\,. \tag{2.11}$$

The combined action, $S_{\text{tot}} := S_{\text{bulk}} + S_{\text{bdry}}$, has a lot of symmetries. In particular, it is covariant under the local geometrical symmetries of the target space, including diffeomorphisms of $\widehat{M}$, and the latter depend on all of the coordinates including the Chan-Paton fermions. There is a U(1) gauge symmetry, $\mathcal{A} \mapsto \mathcal{A} + da$, as well as a manifest abelian one-form gauge symmetry, $B \mapsto B + db$, $\mathcal{A} \mapsto \mathcal{A} + b$, where appropriate pull-backs are understood. The non-abelian gauge symmetry will emerge later from the U(1) and the vertical diffeomorphisms of $\widehat{M}$.

The equation of motion for the boundary fermions, following from (2.9) and (2.11), is

$$\dot{\eta}^{\widehat{\mu}} = -\left(\dot{z}^N K_{N\widehat{\nu}} + \partial_{\widehat{\nu}} z^{\underline{M}} E_{\underline{M}}{}^{\underline{a}} E_{1\underline{a}}\right) N^{\widehat{\mu}\widehat{\nu}}\,, \tag{2.12}$$

where

$$K := d\mathcal{A} - \widehat{f}^* B \tag{2.13}$$

and $N^{\widehat{\mu}\widehat{\nu}}$ is the inverse of $K_{\widehat{\mu}\widehat{\nu}}$. Using this equation in the total boundary variation (again from (2.9) and (2.11)) we find that the residual variation is

$$\delta_{\text{bdry}} S_{\text{tot}} = -\int_{\partial\Sigma} dt\, \delta z^N \left\{\dot{z}^M \left(K_{MN} - K_{M\widehat{\mu}} N^{\widehat{\mu}\widehat{\nu}} K_{\widehat{\nu}N}\right) + \mathcal{D}_N z^{\underline{M}} E_{\underline{M}}{}^{\underline{a}} E_{1\underline{a}}\right\}, \tag{2.14}$$

where we have introduced a covariant derivative (vector field)

$$\mathcal{D}_M := \partial_M - K_M{}^{\widehat{\mu}} \partial_{\widehat{\mu}}; \qquad \text{where } K_M{}^{\widehat{\mu}} := K_{M\widehat{\nu}} N^{\widehat{\nu}\widehat{\mu}}\,. \tag{2.15}$$

The object in the round brackets on the right-hand-side of (2.14) will play a central rôle in the following and so we give it a new name, $\mathcal{F}_{MN}$,

$$\mathcal{F}_{MN} := K_{MN} - K_{M\widehat{\mu}} N^{\widehat{\mu}\widehat{\nu}} K_{\widehat{\nu}N}\,. \tag{2.16}$$

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2.2 Kappa-symmetry

The total action will be required to be invariant under kappa-symmetry. This can be discussed for the bulk and boundary separately. Kappa-symmetry in the bulk is of course well-understood and is ensured by requiring the background superspace to be an arbitrary supergravity background, i.e. the background fields are those of the supergravity multiplet and they are on-shell. It has been discussed for type-II strings in refs [42, 43]. It is the boundary kappa-symmetry which is of most interest in this paper. We shall define a boundary kappa-symmetry transformation to be (the leading component of) an odd diffeomorphism of the worldvolume of the brane, M. This is the usual superembedding interpretation of kappa-symmetry. In order to do this we have to select an odd sub-bundle F of the tangent bundle TM, and to do this we have to make use of a worldvolume supervielbein $E_M{}^A$. We then set

$$\delta_\kappa z^\alpha = v^\alpha \qquad \delta_\kappa z^a = 0\,, \tag{2.17}$$

where $\delta z^A := \delta z^M E_M{}^A$. The odd diffeomorphism parameter v^α can be related to the kappa-symmetry parameter in the usual fashion,

$$\kappa^{\underline{\alpha}} := v^\alpha E_\alpha{}^M \partial_M z^{\underline{M}} E_{\underline{M}}{}^{\underline{\alpha}}\,. \tag{2.18}$$

If we substitute this variation into (2.14) we see that kappa-symmetry on the boundary will be assured provided that two geometrical conditions are satisfied,

$$\mathcal{E}_\alpha{}^{\underline{a}} = 0 \tag{2.19}$$

and

$$\mathcal{F}_{\alpha B} = 0 \tag{2.20}$$

where

$$\mathcal{E}_A{}^{\underline{A}} := E_A{}^M \mathcal{D}_M z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}}\,. \tag{2.21}$$

The equations (2.19) and (2.20) are remarkably similar to the standard equations describing single D-branes in the superembedding formalism. They were derived in a GS approach in [17] and can also be understood in terms of superembeddings of branes ending on branes [18]. The first equation, the superembedding constraint, is now generalised to include the covariant derivative $\mathcal{D}_M$, while the constraint on the two-form will contain information about the non-abelian field strength tensor. As we shall see, these two equations determine the structure of the multiplets on the brane and their equations of motion.

It might seem that the form of the kappa-symmetry constraints depends on the choice of odd tangent bundle on M. This is indeed the case, since if we made a different choice, say $E'_\alpha = E_\alpha + \Lambda_\alpha{}^a E_a$, where Λ is some field, then the form of the constraints would change. A better statement would be to say that kappa-symmetry implies that there exists a choice of odd tangent bundle F on the brane such that equations (2.19) and (2.20) hold.

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3. Geometrical interpretation

In this section we shall give a geometrical interpretation of the results derived above. The first observation is that the vector field $\mathcal{D}_M$ (2.15) defines horizontal subspaces in the tangent spaces of $\widehat{M}$. Thus we see that the use of the equations of motion for the fermions induces a bundle structure in $\widehat{M}$ with a connection defined by the vector field $\mathcal{D}_M$. It should be borne in mind, however, that this structure is not quite the same as the Yang-Mills structure which will be associated with a different covariant derivative (denoted by D_M) and which will be discussed later. Moreover, as we shall see, the horizontal subspaces are not preserved by η-dependent diffeomorphisms of M, so that the bundle structure is in some sense generalised. This generalisation can be thought of as a consequence of maintaining general covariance.

The field $\mathcal{F}_{MN}$ (2.16) is a horizontal two-form which is in fact the horizontal projection of K. The definition of the horizontal subspace makes use of K and it can easily be checked that, in the horizontal lift basis $(\mathcal{D}_M, \partial_{\widehat{\mu}})$, K has either purely horizontal or purely vertical components. The dual basis is

$$dz^M \qquad \text{and} \qquad e^{\widehat{\mu}} := d\eta^{\widehat{\mu}} + dz^M K_M{}^{\widehat{\mu}}\,, \tag{3.1}$$

so that

$$K = \frac{1}{2}\left(dz^N dz^M \mathcal{F}_{MN} + e^{\widehat{\nu}} e^{\widehat{\mu}} K_{\widehat{\mu}\widehat{\nu}}\right). \tag{3.2}$$

The specification of a horizontal subspace allows us to define the notion of a covariant pull-back of forms from $\underline{M}$. This is just the horizontal part of the ordinary pull-back. We shall denote such pull-backs by hats. Later on we shall have a different covariant pull-back which will be denoted by a tilde. For example, given a one-form b on $\underline{M}$ its covariant pull-back is

$$\widehat{b}_M := \mathcal{D}_M z^{\underline{M}} b_{\underline{M}}\,. \tag{3.3}$$

Higher-rank forms will, of course, have mixed components. For the B-field, for example, as well as $\widehat{B}_{MN}$, we have

$$\widehat{B}_{M\widehat{\nu}} := \partial_{\widehat{\nu}} z^{\underline{N}} \mathcal{D}_M z^{\underline{M}} B_{\underline{MN}}\,. \tag{3.4}$$

We shall denote the purely horizontal pull-back of a form by $\widehat{f}$ in contrast to the full pull-back which will be denoted by $\widehat{f}^*$.

The field $\mathcal{F}_{MN}$ obeys a nice Bianchi identity which follows as a consequence of its definition as the horizontal part of K, the fact that K has no mixed component, and the Bianchi identity for K which is

$$dK = -\widehat{f}^* H\,. \tag{3.5}$$

The $\mathcal{F}$ Bianchi is

$$\mathcal{D}\mathcal{F} = -\widehat{f} H\,, \tag{3.6}$$

or, in components,

$$3\mathcal{D}_{[M}\mathcal{F}_{NP]} = -\widehat{H}_{MNP}\,. \tag{3.7}$$

It is instructive to check the consistency of (3.7) directly. This makes use of the curvature of $\mathcal{D}_M$ which is defined by

$$[\mathcal{D}_M, \mathcal{D}_N] := -\mathcal{R}_{MN}{}^{\widehat{\rho}}\,\partial_{\widehat{\rho}} = N^{\widehat{\rho}\widehat{\sigma}}\left(\partial_{\widehat{\sigma}}\mathcal{F}_{MN} + \widehat{H}_{\widehat{\sigma}MN}\right)\partial_{\widehat{\rho}}\,. \tag{3.8}$$

To show the consistency of (3.7) one applies a second covariant derivative to both sides, antisymmetrises and makes use of the definition of the curvature $\mathcal{R}$.

Now a choice of a horizontal subbundle in $T\widehat{M}$ is normally only preserved by diffeomorphisms of the base M which do not depend on the fibre coordinates, but in the present case we will shortly see that the geometrical constraints which follow from kappa-symmetry are covariant under η-dependent diffeomorphisms of M. Let v be a vector field on $\widehat{M}$ which generates an infinitesimal diffeomorphism. It can be written

$$\begin{aligned} v &= v^M\partial_M + v^{\widehat{\mu}}\partial_{\widehat{\mu}} \\ &= \widehat{v}^M\mathcal{D}_M + \widehat{v}^{\widehat{\mu}}\partial_{\widehat{\mu}} \end{aligned} \tag{3.9}$$

from which

$$\begin{aligned} \widehat{v}^M &= v^M \\ \widehat{v}^{\widehat{\mu}} &= v^{\widehat{\mu}} + v^M K_M{}^{\widehat{\mu}}\,. \end{aligned} \tag{3.10}$$

A short computation shows that

$$\delta K_M{}^{\widehat{\mu}} = \widehat{v}^N\left(\mathcal{D}_N K_M{}^{\widehat{\mu}} - \mathcal{D}_M K_N{}^{\widehat{\mu}}\right) - N^{\widehat{\mu}\widehat{\nu}}\partial_{\widehat{\nu}}\widehat{v}^N\mathcal{F}_{NM} + \mathcal{D}_M\widehat{v}^{\widehat{\mu}} + \widehat{v}^{\widehat{\nu}}\partial_{\widehat{\nu}}K_M{}^{\widehat{\mu}}\,. \tag{3.11}$$

The transformation of a horizontal one-form is then found to be

$$\delta\widehat{b}_M = \widehat{v}^N\mathcal{D}_N\widehat{b}_M + \mathcal{D}_M\widehat{v}^N\widehat{b}_N + N^{\widehat{\mu}\widehat{\nu}}\partial_{\widehat{\nu}}\widehat{v}^N\mathcal{F}_{NM}b_{\widehat{\mu}} + \widehat{v}^{\widehat{\mu}}\partial_{\widehat{\mu}}\widehat{b}_M\,. \tag{3.12}$$

The last term on the first line of the right-hand-side of this equation shows that the violation of horizontality under such transformations is caused by the η-dependence of the $\widehat{v}^M$ parameter. However, as we remarked above, the geometrical constraints implied by kappa-symmetry are covariant. It is straightforward to show that $\mathcal{F}$ transforms in a nice tensorial fashion. This follows from the facts that $\mathcal{F}$ is the horizontal part of K and that the latter has no mixed component in the horizontal lift basis. Thus the undesired term involving N which appears in (3.12) does not appear in the transformation of $\mathcal{F}$, which is

$$\delta\mathcal{F}_{MN} = \widehat{v}^P\mathcal{D}_P\mathcal{F}_{MN} + \mathcal{D}_M\widehat{v}^P\mathcal{F}_{PN} - \mathcal{D}_N\widehat{v}^P\mathcal{F}_{PM} + \widehat{v}^{\widehat{\mu}}\partial_{\widehat{\mu}}\mathcal{F}_{MN}\,. \tag{3.13}$$

On the other hand, the generalised superembedding condition (2.19) involves the horizontal pull-back of the supervielbein, as can be seen from equation (2.21). There will therefore be a contribution to the transformation of $\mathcal{E}_A{}^{\underline{A}}$ of the form

$$\delta\mathcal{E}_A{}^{\underline{A}} \sim E_A{}^M N^{\widehat{\mu}\widehat{\nu}}\partial_{\widehat{\nu}}\widehat{v}^N\mathcal{F}_{NM}\partial_{\widehat{\mu}}z^{\underline{M}}E_{\underline{M}}{}^{\underline{A}}\,. \tag{3.14}$$

This looks slightly unpleasant, but under closer inspection we observe that the superembedding condition (2.19) transforms into the $\mathcal{F}$ constraint (2.20), while the latter transforms into itself. This argument makes it plausible that the kappa-symmetry constraints are indeed covariant under diffeomorphisms of $\widehat{M}$, but it is not complete as we have not taken into account the transformation properties of the supervielbein. We shall return to this point in the section on the covariant formalism.

We conclude this section with a discussion of gauge-fixing and the non-abelian gauge field. The transformation of the gauge field under U(1) gauge transformations and diffeomorphisms can be written (in a coordinate basis)

$$\begin{aligned} \delta\mathcal{A}_M &= \partial_M a + v^N\left(\partial_N\mathcal{A}_M - \partial_M\mathcal{A}_N\right) + v^{\widehat{\nu}}\left(\partial_{\widehat{\nu}}\mathcal{A}_M - \partial_M\mathcal{A}_{\widehat{\nu}}\right) \\ \delta\mathcal{A}_{\widehat{\mu}} &= \partial_{\widehat{\mu}} a + v^N\left(\partial_N\mathcal{A}_{\widehat{\mu}} - \partial_{\widehat{\mu}}\mathcal{A}_N\right) + v^{\widehat{\nu}}\left(\partial_{\widehat{\nu}}\mathcal{A}_{\widehat{\mu}} + \partial_{\widehat{\mu}}\mathcal{A}_{\widehat{\nu}}\right), \end{aligned} \tag{3.15}$$

where we have shifted the original U(1) parameter a by $(v^M\mathcal{A}_M + v^{\widehat{\mu}}\mathcal{A}_{\widehat{\mu}})$. Provided that $\partial_{\widehat{\nu}}\mathcal{A}_{\widehat{\mu}} + \partial_{\widehat{\mu}}\mathcal{A}_{\widehat{\nu}}$ is non-singular we can use $v^{\widehat{\mu}}$ to bring $\mathcal{A}_{\widehat{\mu}}$ to a standard form which we shall take to be

$$\mathcal{A}_{\widehat{\mu}} = \frac{1}{2}\eta_{\widehat{\mu}} := \frac{1}{2}\delta_{\widehat{\mu}\widehat{\nu}}\eta^{\widehat{\nu}}\,. \tag{3.16}$$

In this gauge residual vertical diffeomorphisms are given by

$$v^{\widehat{\mu}} = \delta^{\widehat{\mu}\widehat{\nu}}\left(-\partial_{\widehat{\nu}} a + v^M A_{M\widehat{\nu}}\right) \tag{3.17}$$

where

$$A_{M\widehat{\nu}} := \partial_{\widehat{\nu}}\mathcal{A}_M\,. \tag{3.18}$$

We shall now define A_M to be $\mathcal{A}_M$ in this gauge. This is the non-abelian gauge field whose field strength tensor is

$$F_{MN} := \partial_M A_N - \partial_N A_M + (A_M, A_N)\,, \tag{3.19}$$

where the bracket $(,)$ can be thought of as a fibre Poisson bracket and is defined by

$$(f,g) := \delta^{\widehat{\mu}\widehat{\nu}}\partial_{\widehat{\mu}} f\,\partial_{\widehat{\nu}} g\,. \tag{3.20}$$

The gauge transformation of A_M is

$$\delta A_M = \partial_M a + (A_M, a)\,. \tag{3.21}$$

The classical gauge group is thus the symplectic group of the (Grassmann odd) fibres. If we were to quantise the fermions according to the rule $\eta \to \gamma$ we would obtain the corresponding gauge group in the quantum theory and this is the group we are ultimately interested in. For n fermions this group is $U(2^k)$, $k = \frac{1}{2}(n-1)$, n odd, or $U(2^k)\times U(2^k)$, $k = \frac{1}{2}(n-2)$, n even. It is possible to have gauge group $U(2^{\frac{1}{2}n})$ if one allows all powers of η in superfield expansions [40]. Note that the leading abelian gauge field in the η-expansion of A_M is the gauge field for the U(1) factor of the non-abelian group.

Under horizontal diffeomorphisms A_M transforms as

$$\delta A_M = v^N F_{NM}\,. \tag{3.22}$$

If one includes the one-form gauge parameter in this calculation one can also find the corresponding transformation of A; it is

$$\delta A_M = D_M z^{\underline{M}} b_{\underline{M}} := \left(\partial_M z^{\underline{M}} + A_M{}^{\widehat{\mu}} \partial_{\widehat{\mu}} z^{\underline{M}}\right) b_{\underline{M}} . \tag{3.23}$$

In this last equation we have introduced a second covariant derivative

$$D_M := \partial_M + A_M{}^{\widehat{\mu}} \partial_{\widehat{\mu}} , \qquad A_M{}^{\widehat{\mu}} \partial_{\widehat{\mu}} := \delta^{\widehat{\mu}\widehat{\nu}} A_{M\widehat{\mu}} \partial_{\widehat{\nu}} , \tag{3.24}$$

which defines the horizontal subspaces in $T\widehat{M}$ associated with the Yang-Mills connection.

It is now straightforward to relate $\mathcal{F}$ to the Yang-Mills field strength F in the standard gauge. One finds

$$\mathcal{F}_{MN} = F_{MN} - \widetilde{B}_{MN} - \widetilde{B}_{M\widehat{\mu}} N^{\widehat{\mu}\widehat{\nu}} \widetilde{B}_{\widehat{\nu}N} , \tag{3.25}$$

where the tilde denotes a horizontal pull-back constructed using D_M in place of $\mathcal{D}_M$. Note that in the standard gauge (3.16) N is the inverse of

$$K_{\widehat{\mu}\widehat{\nu}} = \delta_{\widehat{\mu}\widehat{\nu}} - B_{\widehat{\mu}\widehat{\nu}} . \tag{3.26}$$

Equation (3.25) gives the generalisation of the modified field strength of a single D-brane in the non-abelian case, at least in this classical approximation.

4. D9-brane

In this section we shall discuss the D9-brane in a flat IIB background. We shall go directly to the physical gauge and compute the Yang-Mills field strength in the standard flat $N = 1, D = 10$ superspace basis. The computation is similar to the abelian case [44, 45]. The only non-zero component of the torsion is

$$T_{\underline{\alpha}\underline{\beta}}{}^{\underline{c}} \to T_{\alpha i \beta j}{}^{c} = -i\delta_{ij} (\gamma^c)_{\alpha\beta} , \tag{4.1}$$

where the notation indicates that the underlined 32-component odd index is replaced by a doublet of 16-component Majorana-Weyl indices. Furthermore, since there are no even directions transverse to the brane we can simply identify the even tangent spaces of M and $\underline{M}$. In the physical gauge we shall identify $\theta^{\alpha 1}$ with the odd coordinate of the brane. The NS three-form H is taken to be

$$\begin{aligned} H &= \frac{1}{2} E^c E^{\underline{\beta}} E^{\underline{\alpha}} H_{\underline{\alpha}\underline{\beta}c} \\ &= -\frac{i}{2} E^c E^{\beta j} E^{\alpha i} (\gamma_c)_{\alpha\beta} (\tau_1)_{ij} \\ &= -i E^c E^{\beta 2} E^{\alpha 1} (\gamma_c)_{\alpha\beta} , \end{aligned} \tag{4.2}$$

where τ_1 is a Pauli matrix and where

$$\begin{aligned} E^{\alpha i} &= d\theta^{\alpha i} , \\ E^a &= dx^a - \frac{i}{2} d\theta^{\alpha i} (\gamma^a)_{\alpha\beta} \theta^{\beta i} \end{aligned} \tag{4.3}$$

are the usual supersymmetric basis one-forms of flat IIB superspace. A permissible choice for the potential two-form B is

$$
\begin{aligned}
B_{a\beta 1} &= i(\gamma_a\theta^2)_\beta \\
B_{\alpha 1\beta 2} &= \frac{1}{3}(\gamma^a\theta^2)_\alpha(\gamma_a\theta^2)_\beta\,,
\end{aligned}
\tag{4.4}
$$

with all other components being zero. Here $(\gamma^a\theta^2)_\alpha = (\gamma^a)_{\alpha\beta}\theta^{\beta 2}$. An advantage of this choice is that this B is manifestly invariant under the first supersymmetry since it does not depend on x or θ^1. This means that we shall not have to include any specific boundary terms in order to ensure that this symmetry holds.

We now come to the gauge-fixing. We shall choose the standard gauge for $\mathcal{A}$, i.e. $\mathcal{A}_{\widehat{\mu}} = \frac{1}{2}\eta_{\widehat{\mu}}$, and we shall also choose the physical gauge for the string sigma model field on the boundary, $z^{\underline{M}}(z,\eta)$. Thus we set

$$
\underline{x}^m = x^m \qquad \text{and} \qquad \underline{\theta}^{\alpha 1} = \theta^\alpha\,,
\tag{4.5}
$$

while

$$
\underline{\theta}^{\alpha 2} = \Lambda^\alpha(x,\theta,\eta)\,.
\tag{4.6}
$$

In these two equations we have underlined the target space coordinates for clarity. The spinor field Λ contains all of the covariant physical fields. Its leading component in the η-expansion corresponds to the U(1) gauge multiplet which describes the motion of the centre of mass. The higher terms will correspond to the other, non-abelian, gauge multiplets.

We note that the pull-back of B in the purely vertical direction vanishes in this gauge. We have

$$
B_{\widehat{\mu}\widehat{\nu}} = (\partial_{\widehat{\nu}}z)^{\underline{B}}(\partial_{\widehat{\mu}}z)^{\underline{A}}B_{\underline{AB}}\,.
\tag{4.7}
$$

However, only θ^2 depends on η, and so $B_{\widehat{\mu}\widehat{\nu}}$ must vanish as $B_{a\beta 2} = B_{\alpha 2\beta 2} = 0$. This in turn implies that

$$
K_{\widehat{\mu}\widehat{\nu}} = \delta_{\widehat{\mu}\widehat{\nu}} \qquad \Rightarrow \qquad N^{\widehat{\mu}\widehat{\nu}} = \delta^{\widehat{\mu}\widehat{\nu}}\,.
\tag{4.8}
$$

We can therefore raise or lower $\widehat{\mu}$ indices using δ without fear of ambiguity. We shall also need the covariant pull-backs of B with respect to D_M in order to compute F_{MN} from (3.25). We can compute these directly in the flat basis $e_A = e_A{}^M\partial_M = (\partial_a, d_\alpha)$ where we have used the notation d_α for the usual flat superspace covariant derivative in order to avoid confusion with the Yang-Mills derivative D_M. We shall denote the non-vanishing pull-backs of B in this basis by $\widetilde{b}_{AB}$ and $\widetilde{b}_{A\widehat{\mu}}$. Thus

$$
\begin{aligned}
\widetilde{b}_{AB} &= (D_B z)^{\underline{B}}(D_A z)^{\underline{A}}B_{\underline{AB}}\,, \\
\widetilde{b}_{A\widehat{\mu}} &= (\partial_{\widehat{\mu}} z)^{\underline{B}}(D_A z)^{\underline{A}}B_{\underline{AB}}\,,
\end{aligned}
\tag{4.9}
$$

where we emphasise that D_A includes the Yang-Mills field, $D_A = e_A{}^M D_M$. A straightforward calculation yields

$$
\widetilde{b}_{\alpha\beta} = \frac{1}{3}D_{(\alpha}\Lambda\gamma^a\Lambda(\gamma_a\Lambda)_{\beta)}\,,
$$

$$\begin{aligned}\widetilde{b}_{a\beta} &= i(\gamma_a\Lambda)_\beta + \frac{1}{6}D_a\Lambda\gamma^b\Lambda(\gamma_b\Lambda)_\beta\,,\\ \widetilde{b}_{ab} &= 0\,.\end{aligned} \tag{4.10}$$

One also finds

$$\begin{aligned}\widetilde{b}_{\alpha\widehat{\mu}} &= \frac{1}{6}\partial_{\widehat{\mu}}\Lambda\gamma^a\Lambda(\gamma_a\Lambda)_\alpha\,,\\ \widetilde{b}_{a\widehat{\mu}} &= 0\,.\end{aligned} \tag{4.11}$$

Our strategy now is to evaluate the non-abelian field strength tensor F_{MN} from (3.25) using the constraints (2.19) and (2.20). These constraints are written with respect to a preferred basis in M which is determined by an induced supervielbein, whereas we want to compute the components of F with respect to the flat basis in $N=1$ superspace. We can always choose a basis E_A of vector fields in M to have the following form,

$$\begin{aligned}E_\alpha &= d_\alpha + \psi_\alpha{}^b\partial_b\,,\\ E_a &= (A^{-1})_a{}^b\partial_b\,.\end{aligned} \tag{4.12}$$

The dual one-form relations are

$$\begin{aligned}E^\alpha &= e^\alpha\,,\\ E^a &= (e^b - e^\beta\psi_\beta{}^b)A_b{}^a\,,\end{aligned} \tag{4.13}$$

where $e^\alpha = d\theta^\alpha, e^a = dx^a - \frac{i}{2}d\theta\gamma^a\theta$. The generalised superembedding matrix (2.21) can be written

$$\mathcal{E}_A{}^{\underline{A}} = E_A{}^M\left(D_M + B_M{}^{\widehat{\mu}}\partial_{\widehat{\mu}}\right)z^{\underline{M}}E_{\underline{M}}{}^{\underline{A}}\,. \tag{4.14}$$

Using (4.12) in (4.14) we find

$$\mathcal{E}_\alpha{}^{\underline{a}} = -\frac{i}{2}(D_\alpha\Lambda\gamma^a\Lambda) + \psi_\alpha{}^b(\delta_b{}^a - \frac{i}{2}D_b\Lambda\gamma^a\Lambda) - \frac{i}{2}E_\alpha{}^M B_M{}^{\widehat{\mu}}\,\partial_{\widehat{\mu}}\Lambda\gamma^a\Lambda\,. \tag{4.15}$$

The last term can be written

$$\begin{aligned}-\frac{i}{2}E_\alpha{}^M B_M{}^{\widehat{\mu}}\partial_{\widehat{\mu}}\Lambda\gamma^a\Lambda &= -\frac{i}{2}\widetilde{b}_\alpha{}^{\widehat{\mu}}\,\partial_{\widehat{\mu}}\Lambda\gamma^a\Lambda\\ &= -\frac{i}{12}\chi_{\widehat{\mu}}{}^a\chi^{\widehat{\mu}b}(\gamma_b\Lambda)_\alpha\,,\end{aligned} \tag{4.16}$$

where we have introduced the abbreviation

$$\chi_{\widehat{\mu}}{}^a := \partial_{\widehat{\mu}}\Lambda\gamma^a\Lambda\,. \tag{4.17}$$

The constraint $\mathcal{E}_\alpha{}^{\underline{a}} = 0$ therefore allows us to solve for ψ,

$$\psi_\alpha{}^a = \frac{i}{2}(D_\alpha\Lambda\gamma^b\Lambda + \frac{1}{6}\chi^{\widehat{\mu}b}\chi_{\widehat{\mu}}{}^c(\gamma_c\Lambda)_\alpha)(\delta_b{}^a - \frac{i}{2}D_b\Lambda\gamma^a\Lambda)^{-1}\,. \tag{4.18}$$

To find A we can choose

$$\mathcal{E}_a{}^{\underline{a}} = \delta_a{}^{\underline{a}}\,. \tag{4.19}$$

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since there are no even transverse directions. From this one immediately finds

$$A_a{}^b = \delta_a{}^b - \frac{i}{2} D_a \Lambda \gamma^b \Lambda \,. \tag{4.20}$$

We now turn to the $\mathcal{F}$ constraint. From equation (3.25) we have

$$f_{AB} = e_B{}^N e_A{}^M \mathcal{F}_{MN} + \widetilde{b}_{AB} + \widetilde{b}_A{}^{\widehat{\mu}} \widetilde{b}_{\widehat{\mu}B} \,, \tag{4.21}$$

where f_{AB} denotes the components of the Yang-Mills field strength in the flat $N = 1$ basis. The first term on the right can be evaluated straightforwardly using (2.20) and (4.13),

$$\begin{aligned} \mathcal{F} &= \frac{1}{2} E^b E^a \mathcal{F}_{ab} \\ &= \frac{1}{2} e^\beta e^\alpha \psi_\alpha{}^a \psi_\beta{}^b A_a{}^c A_b{}^d \mathcal{F}_{cd} \\ &\quad - e^b e^\alpha \psi_\alpha{}^a A_a{}^c A_b{}^d \mathcal{F}_{cd} + \frac{1}{2} e^b e^a A_a{}^c A_b{}^d \mathcal{F}_{cd} \,. \end{aligned} \tag{4.22}$$

Using the fact that $b_{ab} = b_{a\widehat{\mu}} = 0$ we find

$$f_{ab} = A_a{}^c A_b{}^d \mathcal{F}_{cd} \,. \tag{4.23}$$

With the aid of (4.10) we thus have

$$f_{\alpha\beta} = \frac{1}{3} D_{(\alpha} \Lambda \gamma^a \Lambda (\gamma_a \Lambda)_{\beta)} + \psi_\alpha{}^a \psi_\beta{}^b f_{ab} - \frac{1}{36} \chi_{\widehat{\mu}}{}^a \chi^{\widehat{\mu} b} (\gamma_a \Lambda)_\alpha (\gamma_b \Lambda)_\beta \,, \tag{4.24}$$

$$f_{a\beta} = i(\gamma_a \Lambda)_\beta + \frac{1}{6} D_a \Lambda \gamma^b \Lambda (\gamma_b \Lambda)_\beta - \psi_\beta{}^b f_{ab} \,. \tag{4.25}$$

These two equations are the main results for this section. In order to bring them to a slightly more familiar form it is useful to introduce a field $h_\alpha{}^\beta$ which is the counterpart of the h field which arises in superembeddings for single branes. As in the abelian case we can put

$$\mathcal{E}_\alpha{}^{\underline{\beta}} = \begin{cases} \mathcal{E}_\alpha{}^{\beta 1} = \delta_\alpha{}^\beta \\ \mathcal{E}_\alpha{}^{\beta 2} = h_\alpha{}^\beta \end{cases} . \tag{4.26}$$

Using (4.12) and (4.14) we find

$$h_\alpha{}^\beta = D_\alpha \Lambda^\beta + \psi_\alpha{}^b D_b \Lambda^\beta + \frac{1}{6} \chi^{\widehat{\mu} a} \partial_{\widehat{\mu}} \Lambda^\beta (\gamma_a \Lambda)_\alpha \,. \tag{4.27}$$

With the aid of this formula one can show that

$$h_\alpha{}^\beta (\gamma^a \Lambda)_\beta = -2i \psi_\alpha{}^a \,. \tag{4.28}$$

Equation (4.27) can be inverted to give

$$D_\alpha \Lambda^\beta = h_\alpha{}^\gamma \left(\delta_\gamma{}^\beta - \frac{i}{2} (\gamma^a \Lambda)_\gamma D_a \Lambda^\beta \right) - \frac{1}{6} \partial_{\widehat{\mu}} \Lambda^\gamma \partial^{\widehat{\mu}} \Lambda^\beta (\gamma^a \Lambda)_\gamma (\gamma_a \Lambda)_\alpha \,. \tag{4.29}$$

Using (4.28) in (4.24), (4.25) we find

$$f_{\alpha\beta} = \frac{1}{3}D_{(\alpha}\Lambda\gamma^a\Lambda(\gamma_a\Lambda)_{\beta)} - \frac{1}{4}h_\alpha{}^\gamma h_\beta{}^\delta(\gamma^a\Lambda)_\gamma(\gamma^b\Lambda)_\delta f_{ab} - \frac{1}{36}\chi_{\widehat{\mu}}{}^a\chi^{\widehat{\mu}b}(\gamma_a\Lambda)_\alpha(\gamma_b\Lambda)_\beta \tag{4.30}$$

$$f_{a\beta} = i(\gamma_a\Lambda)_\beta + \frac{1}{6}D_a\Lambda\gamma^b\Lambda(\gamma_b\Lambda)_\beta - \frac{i}{2}h_\beta{}^\gamma(\gamma^b\Lambda)_\gamma f_{ab}\,. \tag{4.31}$$

It is easy to check that our results reduce to those of Kerstan's in the abelian case [45]. The formulae for $f_{\alpha\beta}$ and $f_{a\beta}$ are equations (33) and (34) of his paper (there is a factor of $\frac{i}{2}$ missing from the third term in (34)). Berkovits and Pershin [36] also agree with Kerstan in this case.[1] One can transform to their conventions from ours by identifying Λ with $-W$, by replacing γ_a by $-i\gamma_a$ and by adding a minus sign for each commutator. To compare with their results at order η^2 one has to expand our fields to first non-trivial order, so that $\Lambda \sim -W - \frac{1}{2}\eta^2\widehat{W}$. If one looks at their equations for $f_{\alpha\beta}$ and $f_{a\beta}$, their equations (4.6) and (4.7), one sees exactly the same structure as one does by expanding our equations (4.30) and (4.31) out keeping only the η^2 terms. To do this one has to identify h with the object BP refer to as $-\frac{1}{4}(\gamma F)$. Thus our equation (4.29) is similar to (3.4) in BP and gives their (4.7) when expanded to η^2. The only terms which are not quite obvious are the commutator terms in BP. However, it is not difficult to see that these come from the terms in our formalism which have two contracted η derivatives, for example, the third term on the right in (4.30). In fact, we even agree on the coefficients of these terms in both (4.6) (third term on the right in (4.29)) and (4.7) (third term on the right in (4.30)).

5. Covariant formulation

5.1 Generalised superembeddings

In section three we have argued that the geometrical equations which arise from kappa-symmetry should be covariant under the full diffeomorphism group of $\widehat{M}$, although this is not manifest. In this section we shall develop a manifestly covariant formalism for the geometry of $\widehat{M}$ and the map from $\widehat{M}$ to $\underline{M}$ which will allow us to complete the proof of covariance. In order to do this we shall introduce a structure on $\widehat{M}$ which allows us to distinguish horizontal and vertical directions in a covariant way. We then introduce a connection for suitable structure group in a similar fashion to the standard superspace formalism. The resulting geometry is not strictly speaking that of a gauge bundle over the brane worldvolume. Indeed, the vertical (η) distribution is not integrable as it would be if we were reformulating the geometry of a gauge bundle in this way.

We define basis one-forms $\widehat{E}^{\widehat{A}}$ and dual vector fields $\widehat{E}_{\widehat{A}}$ in the usual way by means of a supervielbein and its inverse,

$$\widehat{E}^{\widehat{A}} := dz^{\widehat{M}}E_{\widehat{M}}{}^{\widehat{A}} \qquad \widehat{E}_{\widehat{A}} := E_{\widehat{A}}{}^{\widehat{M}}\partial_{\widehat{M}}\,. \tag{5.1}$$

The structure group is taken to be a product which splits the tangent space into horizontal and vertical components, so that the splitting $\widehat{E}^{\widehat{A}} = (\widehat{E}^A, \widehat{E}^{\widehat{\alpha}})$ is invariant.

[1]The BP result for the D9-brane (up to η^2) has also been derived in a GS calculation starting from the flat target space action [46].

The horizontal part of the structure group, acting on $\widehat{E}^A$, will be the same group as the superspace structure group, while the vertical part will turn out to be $O(n)$ and the parameters of these groups should depend on all of the coordinates. We then introduce a set of connection one-forms $\Omega_{\widehat{A}}{}^{\widehat{B}}$ to allow us to carry out covariant differentiation with respect to the structure group. The torsion and curvature forms are defined in the usual way,

$$\begin{aligned} T^{\widehat{A}} &:= d\widehat{E}^{\widehat{A}} + \widehat{E}^{\widehat{B}}\Omega_{\widehat{B}}{}^{\widehat{A}} \\ R_{\widehat{A}}{}^{\widehat{B}} &:= d\Omega_{\widehat{A}}{}^{\widehat{B}} + \Omega_{\widehat{A}}{}^{\widehat{C}}\Omega_{\widehat{C}}{}^{\widehat{B}} . \end{aligned} \tag{5.2}$$

At this stage we have introduced a lot of new objects and, at the same time, we have enlarged the gauge group, so that it will be necessary to impose some constraints. In the brane context we shall be able to derive many of these from what one might call the generalised superembedding formalism, i.e the geometry of the map $\widehat{f} : \widehat{M} \to \underline{M}$.

To summarise, we now have a superspace $\widehat{M}$, equipped with the above structure, and we also have the two-form K satisfying the Bianchi identity $dK = -\widehat{f}^*H$. We shall impose the following constraints on K,

$$\begin{aligned} K_{\widehat{\alpha}\widehat{\beta}} &= \delta_{\widehat{\alpha}\widehat{\beta}} \\ K_{A\widehat{\beta}} &= 0 . \end{aligned} \tag{5.3}$$

The first of these is natural in the sense that we do not wish to introduce any new fields and because this component of K must be non-singular. It also reflects the odd-symplectic structure of the vertical direction. We shall further suppose that $\delta_{\widehat{\alpha}\widehat{\beta}}$ is covariantly constant so that the vertical structure group is indeed $O(n)$. The second constraint in (5.3) corresponds to the fact that K has no mixed components in the horizontal lift basis. The remaining non-trivial part of K is related to the field $\mathcal{F}_{AB}$ as we shall discuss later. The K Bianchi identity has the following components

$$3\nabla_{[A}K_{BC]} + 3T_{[AB}{}^{D}K_{|D|C]} = -H_{ABC} , \tag{5.4}$$

$$\nabla_{\widehat{\alpha}}K_{BC} + 2T_{\widehat{\alpha}[B}{}^{D}K_{|D|C]} + T_{BC}{}^{\widehat{\delta}}K_{\widehat{\delta}\widehat{\alpha}} = -H_{\widehat{\alpha}BC} , \tag{5.5}$$

$$T_{\widehat{\alpha}\widehat{\beta}}{}^{D}K_{DC} + 2T_{C(\widehat{\alpha}}{}^{\widehat{\delta}}K_{\widehat{\beta})\widehat{\delta}} = -H_{\widehat{\alpha}\widehat{\beta}C} , \tag{5.6}$$

$$3T_{(\widehat{\alpha}\widehat{\beta}}{}^{\widehat{\delta}}K_{\widehat{\gamma})\widehat{\delta}} = -H_{\widehat{\alpha}\widehat{\beta}\widehat{\gamma}} . \tag{5.7}$$

The covariant derivative here is constructed using the connection we have just introduced and the terms on the right-hand-sides are the various components of the pull-back of H in this basis. The first of these equations is equivalent to (3.7), while the second can be rearranged to give

$$T_{AB}{}^{\widehat{\gamma}} = -N^{\widehat{\gamma}\widehat{\delta}}\left(H_{AB\widehat{\delta}} + \nabla_{\widehat{\delta}}K_{AB} + 2T_{\widehat{\delta}[A}{}^{C}K_{|C|B]}\right) , \tag{5.8}$$

where we have denoted the inverse of $K_{\widehat{\alpha}\widehat{\beta}}$ by $N^{\widehat{\alpha}\widehat{\beta}}$, although it is just $\delta^{\widehat{\alpha}\widehat{\beta}}$ in this basis. The left-hand-side of (5.8) is essentially the curvature $\mathcal{R}$ of $\mathcal{D}$ so that the equation is the counterpart of (3.8). Equation (5.6) allows us to solve for $T_{A(\widehat{\beta}\widehat{\gamma})}$ while (5.7) allows us to solve for $T_{(\widehat{\alpha}\widehat{\beta}\widehat{\gamma})}$, (where we have lowered the upper torsion indices using $K_{\widehat{\alpha}\widehat{\beta}}$). The remaining parts of these components of the torsion may be set to zero using the freedom to choose $\Omega_{A\widehat{\beta}\widehat{\gamma}}$ and $\Omega_{\widehat{\alpha}\widehat{\beta}\widehat{\gamma}}$, both of which are antisymmetric on the last two (Lie algebra) indices. It will turn out that all of the remaining components of the torsion which are so far not determined in terms of known quantities can be found from the torsion equation of the generalised superembedding.

One can continue with an analysis of the curvature tensor but we shall not do so in any detail here as it will not be needed in the rest of the paper. However, it is worth making one point which arises because the structure group in the fermionic direction is orthogonal which is symplectic for odd variables. In this case one finds that the torsion constraints do not determine the purely fermionic connection $\Omega_{\widehat{\alpha},\widehat{\beta}}{}^{\widehat{\gamma}}$ completely. The totally antisymmetric part is left over. This should be regarded as being pure gauge, i.e. we should introduce a shift symmetry precisely of this form. If we then set the curvature $R_{\widehat{\alpha}\widehat{\beta},\widehat{\gamma}}{}^{\widehat{\delta}}$ to zero, and use the $O(n)$ gauge symmetry to set the connection to zero, the additional shift symmetry will allow us to preserve the gauge choice without overconstraining the residual gauge parameters.

We now give a brief discussion of the torsion equation for the generalised superembedding. The embedding matrix can be defined, as in the case of a single brane, to be the derivative of the map $\widehat{f}$ in the preferred bases for both the target space and $\widehat{M}$. Thus we have

$$\widehat{E}_{\widehat{A}}{}^{\underline{A}} = \widehat{E}_{\widehat{A}}{}^{\widehat{M}} \partial_{\widehat{M}} z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}} . \tag{5.9}$$

The torsion equation is

$$2\nabla_{[\widehat{A}} \widehat{E}_{\widehat{B}]}{}^{\underline{C}} + T_{\widehat{A}\widehat{B}}{}^{\widehat{C}} \widehat{E}_{\widehat{C}}{}^{\underline{C}} = \widehat{E}_{\widehat{B}}{}^{\underline{B}} \widehat{E}_{\widehat{A}}{}^{\underline{A}} T_{\underline{AB}}{}^{\underline{C}} . \tag{5.10}$$

We can impose some constraints on the superembedding matrix. The first is the constraint (2.19). The remaining parts of $\widehat{E}_A{}^{\underline{A}}$ can be parametrised in a similar fashion to the single brane case. Thus we can take

$$\begin{aligned}
\widehat{E}_\alpha{}^{\underline{a}} &= 0 , \\
\widehat{E}_a{}^{\underline{a}} &= u_a{}^{\underline{a}} , \\
\widehat{E}_\alpha{}^{\underline{\alpha}} &= u_\alpha{}^{\underline{\alpha}} + h_\alpha{}^{\beta'} u_{\beta'}{}^{\underline{\alpha}} , \\
\widehat{E}_a{}^{\underline{\alpha}} &= h_a{}^{\beta'} u_{\beta'}{}^{\underline{\alpha}} ,
\end{aligned} \tag{5.11}$$

where u denotes an element of the target space structure group in the appropriate representation. We shall also impose some constraints on the new components of the superembedding matrix. They are

$$\begin{aligned}
\widehat{E}_{\widehat{\alpha}}{}^{\underline{a}} &= h_{\widehat{\alpha}}{}^{a'} u_{a'}{}^{\underline{a}} , \\
\widehat{E}_{\widehat{\alpha}}{}^{\underline{\alpha}} &= h_{\widehat{\alpha}}{}^{\alpha'} u_{\alpha'}{}^{\underline{\alpha}} .
\end{aligned} \tag{5.12}$$

In (5.12) the $\widehat{\alpha}$ index can be thought of as a derivative in the fermionic direction, so that these equations imply that the superembedding only depends on the η-coordinates

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in the transverse directions. This is necessary in order to get the right number of degrees of freedom on the brane. In the discussion of section three it was stated that the latter was a result of the η-dependence of the diffeomorphisms of M, but the situation is slightly different in this version as we have introduced new components for the supervielbein which "use up" some of the gauge transformations so that new constraints have to be imposed. The fields $h_\alpha{}^{\alpha'}$ and $h_a{}^{\alpha'}$ can be thought of as M derivatives of the transverse fermion field. The first is related to the field-strength $\mathcal{F}_{ab}$ (or K_{ab}) by the $\mathcal{F}$ Bianchi identity.

As in the standard superembedding formalism the connection $\Omega_{\widehat{A},B}{}^C$ can be found by introducing the Lie algebra-valued one-form $X_{\widehat{A}}$,

$$X_{\widehat{A}} := (\nabla_{\widehat{A}} u)u^{-1}\,, \tag{5.13}$$

and setting some parts of it to zero. That is, we put

$$X_{\widehat{A},b}{}^c = X_{\widehat{A},b'}{}^{c'} = X_{\widehat{A},\beta}{}^\gamma = X_{\widehat{A},\beta'}{}^{\gamma'} = 0\,. \tag{5.14}$$

These equations determine the connection $\Omega_{\widehat{A},B}{}^C$. The connection $\Omega_{\widehat{A},\widehat{\beta}}{}^{\widehat{\gamma}}$ is determined by the torsion constraints discussed above, apart from the totally antisymmetric part of $\Omega_{\widehat{\alpha},\widehat{\beta}\widehat{\gamma}}$ which can be set to zero as explained previously. The torsion equations can then be solved systematically to determine the remaining components of the torsion. In addition one finds constraints on the physical fields and their derivatives. As there are no auxiliary fields and we have manifest supersymmetry it follows that these equations determine the dynamics of the brane multiplet.

5.2 Proof of covariance

The formalism just introduced is manifestly covariant by construction, but it is not quite so obvious how it is related to the earlier geometrical description given in section 3. In the new formalism we have components of the supervielbein in the boundary fermion directions which were not present before, and one might wonder how the old formalism could give covariant results. In particular, the definition of the embedding matrix in the new formalism differs from the earlier one. In order for them to be the same (for $\widehat{A} = A$) one would have to identify $E_A{}^{\widehat{\mu}}$ with $-E_A{}^M K_M{}^{\widehat{\mu}}$, but this is not the case as we shall now discuss in more detail.

We set

$$\begin{aligned} E_{\widehat{\alpha}}{}^M &= E_{\widehat{\alpha}}{}^{\widehat{\mu}} \phi_{\widehat{\mu}}{}^M \\ E_A{}^{\widehat{\mu}} &= E_A{}^M \psi_M{}^{\widehat{\mu}}\,. \end{aligned} \tag{5.15}$$

Then the constraint $K_{\widehat{\alpha}B} = 0$ can be written

$$K_{\widehat{\alpha}B} = E_B{}^N E_{\widehat{\alpha}}{}^{\widehat{\mu}} \left(K_{\widehat{\mu}N} - \psi_N{}^{\widehat{\nu}} K_{\widehat{\nu}\widehat{\mu}} + \phi_{\widehat{\mu}}{}^M (K_{MN} + \psi_N{}^{\widehat{\nu}} K_{M\widehat{\nu}}) \right) = 0\,. \tag{5.16}$$

This can be solved to give

$$\psi_M{}^{\widehat{\mu}} = -K_M{}^{\widehat{\mu}} + \Phi^{\widehat{\mu}N} \mathcal{F}_{NM}\,, \tag{5.17}$$

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where $\mathcal{F}_{MN}$ is defined in (2.16) and

$$\Phi^{\widehat{\mu}N} := (\delta^{\widehat{\mu}}{}_{\widehat{\nu}} + \phi^{\widehat{\mu}N} K_{N\widehat{\nu}})^{-1} \phi^{\widehat{\nu}N} \,. \tag{5.18}$$

A similar calculation for K_{AB} yields

$$K_{AB} = E_B{}^N E_A{}^M (\mathcal{F}_{MN} + \mathcal{F}_{MP} \Phi^{PQ} \mathcal{F}_{QN}) \,, \tag{5.19}$$

where

$$\Phi^{PQ} := \Phi^{\widehat{\mu}P} K_{\widehat{\mu}\widehat{\nu}} \Phi^{\widehat{\nu}Q} \,. \tag{5.20}$$

The horizontal projection of the superembedding matrix can be written as

$$\begin{aligned} \widehat{E}_A{}^{\underline{A}} &= E_A{}^{\widehat{M}} \partial_{\widehat{M}} z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}} \\ &= E_A{}^M (\partial_M + \psi_M{}^{\widehat{\mu}} \partial_{\widehat{\mu}}) z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}} \\ &= E_A{}^M (\mathcal{D}_M + \Phi^{\widehat{\mu}N} \mathcal{F}_{NM} \partial_{\widehat{\mu}}) z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}} \,, \end{aligned} \tag{5.21}$$

so that the difference between the old and new definitions resides in the term with $\mathcal{F}$. Explicitly,

$$\widehat{E}_A{}^{\underline{A}} = \mathcal{E}_A{}^{\underline{A}} + E_A{}^M \Phi^{\widehat{\mu}N} \mathcal{F}_{NM} \partial_{\widehat{\mu}} z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}} \,. \tag{5.22}$$

We also have

$$K_{\alpha B} = \mathcal{F}_{\alpha C} (\delta^C{}_B + \Phi^{CD} \mathcal{F}_{DB}) \,. \tag{5.23}$$

Therefore the original kappa-symmetry constraints

$$\mathcal{E}_\alpha{}^{\underline{a}} = \mathcal{F}_{\alpha B} = 0 \tag{5.24}$$

are precisely equivalent to the manifestly covariant constraints

$$\widehat{E}_\alpha{}^{\underline{a}} = K_{\alpha B} = 0 \,. \tag{5.25}$$

This discussion makes it clear that the truly covariant U(1) field strength is K rather than $\mathcal{F}$. As we have seen the constraints $K_{\alpha B} = 0$ are equivalent to the constraints $\mathcal{F}_{\alpha B} = 0$, but the invariantly defined non-vanishing components are K_{ab} rather than $\mathcal{F}_{ab}$.

6. Discussion

In this paper we have presented a preliminary study of a possible approach to finding the non-abelian counterpart to the supersymmetric Born-Infeld plus Wess-Zumino term action which describes the low energy dynamics of single D-branes [47–51]. The next step in the programme would be to quantise the boundary fermions. If it is possible to do so consistently, one would then arrive at the desired theory which would presumably represent a well-defined approximation to the effective action (or equations of motion) of a set of coincident D-branes. The full effective action would include higher derivative contributions which in our approach should correspond to the quantisation of the bulk part of the action. Since we do not know how to do this for the GS action, it would be necessary to use the pure spinor formalism for this purpose.

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At the level at which we are working, however, the GS/superembedding approach has the advantage that all the symmetries are manifestly realised. In particular, we have been able to achieve gauge invariance for the B-field as well as kappa-symmetry. As we noted in the introduction, previous attempts in this direction have not been completely successful, but in our approach kappa-symmetry with a parameter which depends on the boundary fermions comes out naturally. In the quantised version this would correspond to a matrix-valued kappa parameter as advocated in [19]. As we have seen, the way that kappa-symmetry works is quite complicated; the use of the equations of motion for the boundary fermions introduces a vertical/horizontal splitting in $\widehat{M}$ as a consequence of which the covariance of the kappa-symmetry contraints under diffeomorphisms of M which depend on the boundary fermions is not at all guaranteed. The fact that everything works out consistently is a strong indication in favour of the formalism. It would be interesting to investigate if the pure spinor string with boundary fermions could be analysed in a covariant fashion with nilpotence of the BRS operator as a replacement for kappa-symmetry. Since this can be done for type-II supergravity [52] it should also be possible to do it for this case .

In the classical approximation it makes sense to investigate the equations of motion order by order in the boundary fermions. We have not done this in detail here, postponing a discussion until the fermions are quantised, but we have seen that the equations for the non-abelian $N = 1, D = 10$ superspace field strength for the D9-brane in a flat background (which imply the equations of motion for the component fields) agree with the results of reference [36]. In the abelian case formally similar-looking constraints are known to describe supersymmetric Born-Infeld theory and so these equations must describe a non-abelian version, although it is not so easy to verify this explicitly. We also note that it is straightforward to adapt the D9-brane analysis given here to the case of lower-dimensional branes in flat backgrounds. One of course sees the emergence of non-abelian scalars, here represented as transverse scalars depending on the boundary fermions. There are many complicated interaction terms encoded in our formalism and we would therefore expect to see, for example, the interactions responsible for the dielectric brane phenomenon appearing at some point.

Another way of investigating such terms would be via an action. GS actions for D-branes can be constructed by a systematic procedure in the superembedding formalism starting from a closed $(p+2)$-form on the superworldvolume [53, 54]. This form is directly related to the WZ term but also encodes the kinetic term via supersymmetry. In the approximation we are using in the current paper it is not clear that it makes sense to write an action. However, it should be possible to investigate whether there are candidate $(p + 2)$-forms which could yield action forms after quantisation. For a set of D0-branes we need a closed two-form on M in order to implement the brane action principle. This suggests that we should be looking for a covariantly closed horizontal two-form, $\mathcal{W}$ say, which satisfies an equation of the form

$$(D\mathcal{W})_{ABC} = 0\,, \tag{6.1}$$

where D is a suitably defined covariant exterior derivative which should not involve torsion terms with any $\widehat{\alpha}$-type indices. To get the action form itself one should quantise and then take the trace over the gauge indices in $\mathcal{W}$ in which case we should obtain a closed two-form on M as required. We expect that $\mathcal{W}$ should be constructed from the IIA RR field strengths G_p, $p = 2, 4, \ldots$, which satisfy $dG_{p+2} = H \wedge G_p$ ($G_0 = 0$), and from K_{AB}. For the D0-brane things are slightly simpler since $K_{AB} = H_{ABC} = 0$. If we start with G_2 we find

$$\nabla_{[A}G_{BC]} + T_{[AB}{}^{D}G_{|D|C]} = -T_{[AB}{}^{\widehat{\delta}}G_{\widehat{\delta}C]}\,. \tag{6.2}$$

The term on the right gets in the way, but can be manipulated if we use $dK = 0$ to replace $T_{AB}{}^{\hat{\delta}}$ by $-N^{\hat{\delta}\hat{\epsilon}}H_{AB\hat{\epsilon}}$. We then see that the right-hand side resembles $N^{\hat{\delta}\hat{\epsilon}}(G_2 \wedge H)_{ABC\hat{\delta}\hat{\epsilon}}$. But $G_2 \wedge H = dG_4$, and so we can deal with such a term by appropriately amending G_2. In this way we are naturally led to consider the following two-form as a candidate $\mathcal{W}$,

$$\begin{aligned}\mathcal{W}_{AB} &= G_{AB} - \frac{1}{2}N^{\widehat{\alpha}\widehat{\beta}}G_{\widehat{\alpha}\widehat{\beta}AB} + \frac{1}{8}N^{\widehat{\alpha}\widehat{\beta}}N^{\widehat{\gamma}\widehat{\delta}}G_{\widehat{\alpha}\widehat{\beta}\widehat{\gamma}\widehat{\delta}AB} - \cdots \\ &= \left(\exp(-\frac{1}{2}i_N)\left(\sum G\right)\right)_{AB}\,,\end{aligned} \tag{6.3}$$

where i_N denotes contraction with the bivector $N^{\hat{\alpha}\hat{\beta}}$. This expression is exactly what one would expect from the Wess-Zumino term in the bosonic non-abelian D-brane action since two $\widehat{\alpha}$-type indices on a pulled-back RR form contracted with N correspond to the ι_{ϕ^2} operation of [7], although in our case there will be fermion contributions as well. However, with this definition of $\mathcal{W}$ we do not quite get an equation of the form of (6.1) since there are additional terms left over. These seem to give rise to a divergence in the fermionic direction which one might expect should vanish in the action since gauge invariance corresponds to η-independence at the classical level. We postpone a full discussion of this point until we have quantised the theory, but it is encouraging that the modifed Wess-Zumino term does seem to emerge from supersymmetry. It would also be interesting to see if kappa-symmetry implies any modifications to the interactions found in [7−9] such as those discussed in references [34, 35].

We conclude with a few comments on quantisation. As noted in the introduction, the dependence of the bulk fields on the boundary fermions means that it is not straightforward to integrate out the latter from the path integral. One possibility would be to find an effective boundary lagrangian that would reproduce the desired boundary equations of motion, but it is not clear that such an object exists. Another possibility would be to try deformation quantisation and demand consistency with the symmetries of the problem. There is a natural bracket in the bounday fermion direction given by $N^{\widehat{\alpha}\widehat{\beta}}$ which has the advantage of being covariant. However, it is not Poisson which suggests that it might be appropriate to consider a non-associative star product. This has been advocated in the bosonic sector in the presence of a non-trivial spacetime B-field [55], although here it might be necessary even in a flat background since the superspace B-field is never trivial.

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